Modal Analysis of System Partitioning in Distributed Real-Time Simulations

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***Abstract*—Selection of a suitable decoupling point for system partitioning represents one of the challenges in distributed real-time simulation (DRTS) and (distributed) power hardware in the loop (D-PHIL) setups. In case of DRTS, the decoupling point represents the point where system is partitioned between two digital real-time simulators. In D-PHIL setups, the decoupling point is a point where a device under test is connected (coupled) to the rest of the simulated system. This work proposes a methodology based on the analysis of eigenvalues and participation matrix of the monolithic continuous model of the system, which refers to naturally coupled system, to determine the suitable decoupling point for system partitioning with respect to the simulation fidelity. Sampling period and delay between subsystems are considered in the analysis, and decoupling points analysis is influenced by them. The methodology is validated based on time-domain simulations.**

***Index Terms*—distributed simulations, real-time simulations, participation factors, system partitioning, modal analysis, eigenvalue analysis**

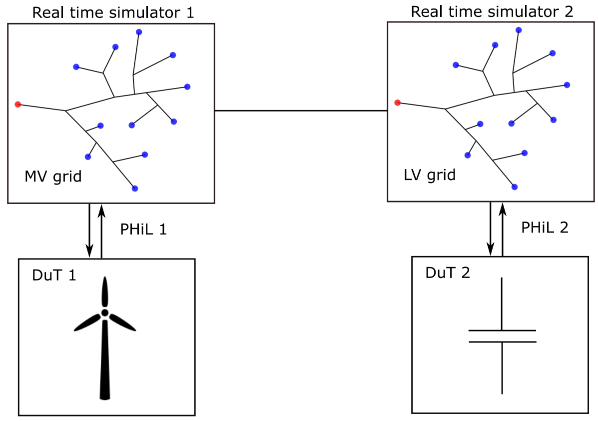

Fig. 1. Distributed PHiL - simple example

I. Introduction

Increased usage of power electronics and renewable energy sources in modern power system grids posed the need for more real-time simulation and Hardware-in-the-Loop (HIL) studies to validate novel concepts and control and protection devices [1]. A natural extension of a single PHiL setup is connection and integration of multiple PHiL test benches hosted in geographically dispersed places [2]. This kind of distributed setup represents distributed PHiL (D-PHiL) simulation. One example of distributed PHiL simulation is illustrated in the Figure 1, and improved version of algorithm presented can be implemented in these systems.

One of the mayor challenges in distributed real-time simulations is partitioning of a monolithic model while ensuring simulation stability and required level of simulation fidelity. Various methods have been proposed in literature for system partitioning in different context. In the context of parallel computing [3], partitioning is optimized with respect to CPU load computation as well as communication between CPUs. With respect to multi-rate simulation, circuit latency is exploited to determine subsystems suitable for small and large simulation time step of multi-rate simulation [4], [5].

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The focus of this work is in particular on distributed real-time simulations and D-PHIL where interfaces between subsystem partitions incorporate communication time delay and often lower sampling period compared to the simulation time step. Thus, criteria for system partitioning in this context differ from other applications, such as parallel computing. In this paper modal analysis is applied for system partitioning on LTI (linear time-invariant) systems with respect to system dynamics and interactions between subsystems considering communication delay and sampling period of the interface.

II. Modal analysis

A systematic procedure for assigning state variables and finding the dynamical equations can be found in [6], if a dynamic system is linear time-invariant electric network. In case of LTI systems, system can be defined by set of equations written in state space form as:

$$\begin{aligned} \dot{\boldsymbol{x}} &= A \cdot \boldsymbol{x} + B \cdot \boldsymbol{u} \\ \boldsymbol{y} &= C \cdot \boldsymbol{x} + D \cdot \boldsymbol{u} \end{aligned} \tag{1}$$

The poles of system are the eigenvalues of the matrix A. These poles can be real and complex, and real part of all system eigenvalues needs to be negative, in order to ensure stability of the system. If A is real, complex eigenvalues appear in conjugate pairs representing one mode. Eigenvalue of matrix A is scalar parameter λ that fulfill equation:

$$A \cdot \boldsymbol{\phi} = \lambda \cdot \boldsymbol{\phi} \tag{2}$$

were $\boldsymbol{\phi}$ is n by 1 vector. To find eigenvalues we can write (2) in form $(A - \lambda \cdot I) \cdot \boldsymbol{\phi} = 0$, and for a non-trivial solution:

$$det(A - \lambda \cdot I) = 0 \tag{3}$$

Equation (3) is the characteristic equation with n solutions that are eigenvalues of A. For any eigenvalue λ_i, the n-column vector $\boldsymbol{\phi_i}$ that satisfies:

$$A \cdot \lambda_i = \lambda_i \cdot \boldsymbol{\phi_i} \tag{4}$$

is the (right) eigenvector of A connected with λ_i. If $\boldsymbol{\phi_i}$ is right eigenvector than $k \cdot \boldsymbol{\phi_i}$ is as well, where k is scalar. Similarly, n-row vector $\boldsymbol{\psi_i}$ that satisfies:

$$\boldsymbol{\psi_i} \cdot A = \lambda_i \cdot \boldsymbol{\psi_i} \tag{5}$$

is left eigenvector connected with λ_i. In order to express the eigenproperties of A, the modal matrices are introduced:

$$\begin{aligned} \Psi &= \left[\boldsymbol{\psi_1^T} \quad \boldsymbol{\psi_2^T} \ldots \boldsymbol{\psi_n^T}\right]^T \\ \Phi &= \left[\boldsymbol{\phi_1} \quad \boldsymbol{\phi_2} \ldots \boldsymbol{\phi_n}\right] \end{aligned} \tag{6}$$

A. *Analysis of eigenvalues*

We are considering just stable systems for our test case analysis, and therefore all the real parts of eigenvalues are negative. Real eigenvalue $\lambda = \sigma$ corresponds to non-oscillatory mode. Time constant of transient response decay of real eigenvalue (mode) is $-\frac{1}{\sigma}$. Each complex conjugate pair of eigenvalues is associated to one complex conjugate mode. For complex pair of eigenvalues $\lambda = \sigma \pm i \cdot \omega$, frequency of oscillation is $\frac{\omega}{2\pi}$ and damping ratio is $-\frac{\sigma}{\sqrt{\sigma^2+\omega^2}}$. Damping ratio determines rate of transient response decay of the amplitude of oscillation of complex conjugate mode. The time constant of transient response of amplitude decay of osculations for complex conjugate mode is $-\frac{1}{\sigma}$.

B. *Participation factors*

The right eigenvector represents activity of the state variables when particular mode is excited, where magnitudes of elements in $\boldsymbol{\phi_i}$ represent degree of activities of n state variables in the ith mode, and angles of elements show phase shifting of the state variables with regard to the mode. Left eigenvector gives which combination of state variables displays the ith mode. Doing analysis of connection of system modes and system states looking individually at right and left eigenvectors can cause a problem because of scaling and units of state variables (elements of right and left eigenvectors depend on units and scaling of state variables). To overcome this issue, participation matrix [7] is introduced.

$$P = [\boldsymbol{p_1} \quad \boldsymbol{p_2} \quad \boldsymbol{p_3} \quad \ldots \quad \boldsymbol{p_n}] \tag{7}$$

with

$$\boldsymbol{p_i} = \begin{bmatrix} p_{1i} \\ p_{2i} \\ \vdots \\ p_{ni} \end{bmatrix} = \begin{bmatrix} \phi_{1i} \cdot \psi_{i1} \\ \phi_{2i} \cdot \psi_{i2} \\ \vdots \\ \phi_{ni} \cdot \psi_{in} \end{bmatrix} \tag{8}$$

where ϕ_{ki} is the kth element of right eigenvector $\boldsymbol{\phi_i}$ and ψ_{ik} is the kth element of left eigenvector $\boldsymbol{\psi_i}$. Element $p_{ki} = \phi_{ki} \cdot \psi ik$ is called participation factor. Influence of specific eigenvalues (poles or modes) on states of the system can be measured using participation matrix. Participation matrix analysis in this paper is performed on the monolithic system model in continuous time domain. Since eigenvectors of the system are the same in continuous and discrete time domain, therefore participation matrix is as well. We assume that simulation time step is small enough comparing to time constants of the system so that discretization does not influence response of the modes and continuous eigenvalues can be considered without losing correctness.

III. Application of Modal Analysis for System Partitioning

This section introduces a methodology for assessment of decoupling points for system partitioning in terms of fidelity and stability of distributed simulation based on modal analysis of monolithic model, in the literature as well called naturally coupled system (NCS). The ideal transformer model (ITM) is assumed to be used as an interface algorithm as represented in Figure 2, and all the conclusions are derived assuming ITM. Assumption made is that the voltage and current measurements have no errors. A variable that is transferred from one subsystem to another is referred to as an interface quantity. The interface quantity is sampled and delayed before being applied on the receiving subsystem. The sampling period is the time difference between two consecutive sending samples of interface quantity from one subsystem to another.

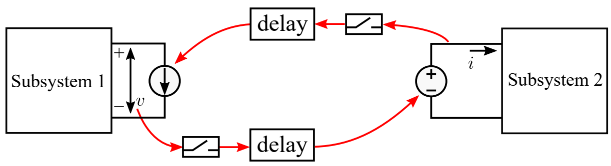

Fig. 2. Distributed simulation or PHiL

Modal analysis for the system partitioning proposed in this paper includes three parts. First, eigenvalues analysis is performed. As a second part participation matrix of monolithic model of the system needs to be defined. Finally, in the third part, decoupling points are analyzed. The following sections introduce the three parts of the methodology for analysis of system partitioning.

A. *Eigenvalues analysis*

In this work is assumed that the monolithic model is stable. Thus, all eigenvalues are located in the left half of s-plane. Critical time constant of the pole (mode) is simulation time step needed to correctly simulate dynamics of that pole (mode). If mode is real ($\lambda = \sigma$), transient response of the pole decays to 37% of the initial amplitude in one time constant $-\frac{1}{\sigma}$, and it is supposed that transient response is decayed in 5 time constants, thus in $-\frac{5}{\sigma}s$. To correctly simulate dynamics of this pole, it is assumed that at least 10 samples of the mode transient response are needed, therefore time step smaller than $-\frac{1}{2\sigma}$ is required, which represents critical time constant of real pole. Critical time constant can be found in the same manner for complex-conjugate eigenvalues pair/mode ($\lambda = \sigma \pm i \cdot \omega$). Time step needed to simulate transient response amplitude decay of this pole is $-\frac{1}{2\sigma}$. Following the rule of the thumb, simulation time step needed to capture oscillations of the pole is 10 times smaller than period of oscillation of the mode (eigenvalue) [8], which results in $\frac{\pi}{5\omega}$. Smaller value of these two time steps represent critical time constant of complex-conjugate mode. Next, modes are characterized as slow or fast based on time delay and sampling period of interest as it is explained in Section IV.

B. Participation matrix analysis

Participation matrix is determined as described in Section II-B. Participation matrix indicates the association of the modes identified in the previous section to the states of the system. Larger participation factor indicates more significant influence of the mode on the state. Small participation factor indicates that influence of the mode on the state can be neglected. Association of the modes to the states is important aspect to be considered for system partitioning, as it indicates state dynamics, further explained in III-C.

C. Decoupling point analysis

Decoupling point analysis is performed for every feasible decoupling point of the circuit. With respect to the selected decoupling point, coupling mode is a mode that influences states in both subsystems, and local mode is a mode that influences states just in one of the subsystems. First, it needs to be determined if partitioned system is loosely coupled for this coupling point. Partitioned system is defined as loosely coupled if all modes are local modes for the selected decoupling point and coupling modes do not exist. If the system is loosely coupled, it should be determined if there is an interface quantity that depends on states influenced by fast local modes. In test cases analyzed in this paper, interface quantity always depends only on one state and directly represents an interface state. If an interface state is influenced by fast local mode, the simulation fidelity can be degraded. Namely, empirical studies performed show that in such cases artificial dynamic is introduced for sampling period larger than the critical time constant of the fast mode influencing interface state. Artificial dynamic represents the dynamic that does not exist in monolithic model as presented in Figure 8. If partitioned system is not loosely coupled for the selected decoupling point, it should be determined if there are fast coupling modes. Recommended time delay, under assumption that sampling period equals one simulation time step, is smaller than critical time constant of fast coupling mode to ensure stability. With respect to the sampling period, under assumption that the time delay is of value of one simulation time step, margin for ensuring stability is larger, and recommended sampling period should be smaller than $1.1 \cdot T_c$ to ensure stability.

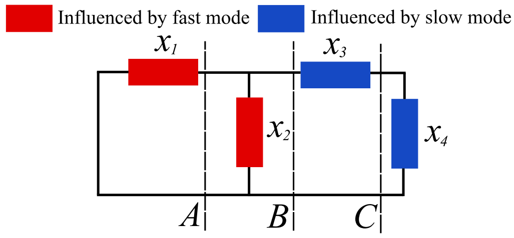

Fig. 3. Test case 2 - Participation of modes in states

Let us consider an example of a system with four states as illustrated in the Figure 3. In this example, two states are influenced by fast and other two states by slow mode of the system. In this test case there are three possible decoupling points: A, B, and C. For decoupling point A, a fast coupling mode exists and it is expected to be the decoupling point with the lowest degree of simulation fidelity compared to the decoupling points B and C. In paper, simulation fidelity refers to how accurately the system response obtained in distributed simulation represents the system response of the monolithic simulation. Namely, in case of decoupling point A, the system is not loosely coupled and there is a fast coupling mode. For decoupling point B, system is loosely coupled but interface quantity (state x_2) is influenced by the fast local mode. For decoupling point C, system is not loosely coupled but, opposite to the case for coupling point A, the coupling mode is slow. The methodology proposed in this work suggests that in addition to analyzing participation matrix and exploiting points of loose coupling, dynamic of the interface quantities and coupling modes should be considered. Namely, empirical analysis indicates that a decoupling point can be a suitable candidate from perspective of exploitation of loose coupling, but one can not say that this point provides higher degree of fidelity than a decoupling point with a coupling mode that is slow. In such cases, additional analysis is required. Therefore, for the test case illustrated in the Figure 3, additional analysis should be performed to determine if decoupling point B or C provides higher degree of fidelity. With increased delays and sampling periods it can happen that fidelity is even better for system partitioned with decoupling point C. Further analysis of circuit in Figure 3 is provided in IV-A2.

The described methodology provides relative assessment of decoupling points for system partitioning based on eigenvalues analysis and participation matrix. Applicability of this method is broad. It can be implemented for analysis of benchmark grid models (e.g. IEEE and CIGRE benchmark models) to determine suitable decoupling points for distributed simulation, PHiL and D-PHiL. Power system simulation solvers based on state space models can automatically generate eigenvalues and state matrix or state matrix of the system can be generated from nodal matrix [5]. Methodology of system partitioning is exemplified based on time-domain simulations and applied on two test cases in IV.

IV. Analysis of simple electrical circuits

A digital real-time simulator uses a fixed-step solver, and therefore simulation time step should be small enough to be able to simulate the desired dynamics. Common simulation time step in electromagnetic transient programs (EMTPs) is 50 µs, with which up to $2-3$ kHz can be simulated based on a common rule of thumb. To correctly simulate system response, rule of the thumb suggests time step size of 10 times smaller value of the period of the fastest frequency following disturbance [8]. In this work, the dynamics of interest are considered the same as in EMTP-based simulation. In addition, the dynamics of interest are divided into fast and slow regions. Definition of fast and slow dynamics (modes/eigenvalues) in method proposed in the paper is defined with respect to maximum sampling period and time delay of interest. Simulation time step for test cases in this work is $\Delta t = 50$ µs. Delay and sampling period of interest are defined as $k_d \cdot \Delta t$ and $k_{sp} \cdot \Delta t$, respectively. For the test cases in this work, $k_{sp} = k_d = k = 25$ is defined. Slow modes are modes with critical time constant larger than $k \cdot \Delta t = 1.25$ ms. Similarly, a fast mode is a mode with critical time constant smaller than $k \cdot \Delta t = 1.25$ ms.

A. Test case 1

A simple electrical circuit, illustrated in Figure 4, is selected as test case 1 to demonstrate the method described in Section III.

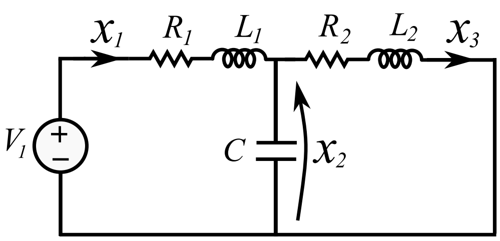

Fig. 4. Test Case 1 - simple electrical circuit

Two sets of parameters that result in different system dynamics are considered and are chosen to give us interesting dynamics for analysis. Simulations of the circuits were running 2 s, and transient process was initiated with changing the input V_1 from 5 to 4 V at 1 s. Parameters, analysis of system partitioning and simulation results for test case 1 are given in the sections IV-A1 and IV-A2.

1) Test case 1.1: Parameters of the circuit for the test case 1.1 and resulting modes are given in the Table I. The system is characterized by one fast $\lambda_{1,2}$ mode and one slow mode λ_3.

TABLE I
Parameters for test case 1.1

Test case	Parameters	Modes		
		eigenvalues	T_c [ms]	type
1.1	$R_1 = 1.0\Omega$ $L_1 = 1.0mH$ $C = 10.0mF$ $R_2 = 1.0\Omega$ $L_2 = 10.0mH$	$\lambda_{1,2} = -495.0 \pm i \cdot 869.01$	0.72	fast
		$\lambda_3 = -200.0$	2.5	slow

Participation matrix for test case 1.1 is:

$$P = \begin{bmatrix} 0.4948 + 0.2934 \cdot i & 0.4948 - 0.2934 \cdot i & 0.0104 \\ 0.5050 - 0.2875 \cdot i & 0.5050 + 0.2875 \cdot i & -0.0100 \\ 0.0002 - 0.0059 \cdot i & 0.0002 + 0.0059 \cdot i & 0.9996 \end{bmatrix}$$

Participation matrix indicates that fast mode $\lambda_{1,2}$ is mainly associated with states x_1 and x_2, while its relation with the state x_3 can be neglected. Similarly, slow mode λ_3 has negligible influence on states x_1 and x_2 and from participation matrix it can be concluded that it mainly influences state x_3. Figure 5 illustrates the described participation of modes in states.

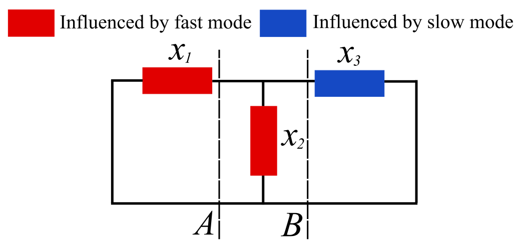

Fig. 5. Test case 1.1 - Participation of modes in states

The main characteristics of decoupling point A are listed below:

- Partitioned system is not loosely coupled
- There is a fast coupling mode with critical time constant of $T_c = 0.72$ ms
- Recommended limit for maximum time delay to ensure stability is $T_c = 0.72$ ms (if sampling period is equal to one time step — 0.05 ms)
- Recommended limit for maximum sampling period to ensure stability is $1.1 \cdot T_c = 0.795$ ms (if time delay is equal to one time step — 0.05 ms)

The main characteristics of decoupling point B are given below:

- Partitioned system is loosely coupled
- There is a local fast mode with critical time constant of $T_c = 0.72$ ms that influences interface state (interface quantity)
- Recommended limit for maximum sampling period to avoid artificial dynamic in system response is $T_c = 0.72$ ms

Following the methodology described in the section III, the highest degree of fidelity when partitioning the circuit illustrated in the Figure 4 is expected to be provided by decoupling point B. Namely, decoupling point B results in a loosely coupled system and none of the modes is a coupling mode. Figure 6 shows time domain response of x_2 and confirms that higher degree of simulation fidelity is achieved when decoupling point B is used for system partitioning.

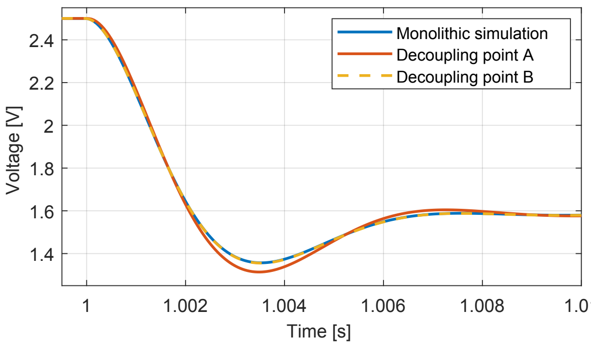

Fig. 6. Test case 1.1: time domain response of x_2 with co-simulation interface of sampling period 0.05 ms, and time delay of 0.05 ms

As decoupling point A results in a partitioned system with a fast coupling mode, simulation fidelity is significantly degraded when interface time delay is increased. For interface time delay of 0.5 ms, x_2 exhibits oscillatory response of very low degree of simulation fidelity as shown in figure 7. For interface time delay of 0.75 ms the simulation is not stable, which confirms that time delay should be smaller than critical time constant $T_c = 0.72$ ms of fast coupling mode in case of decoupling point A.

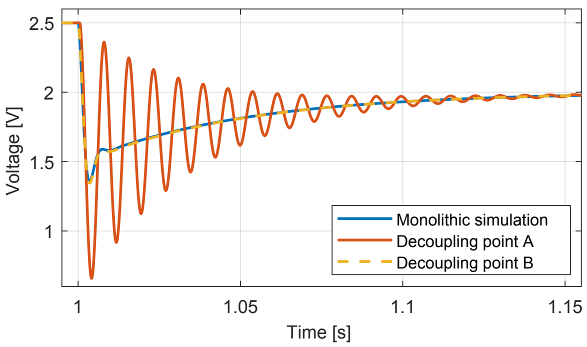

Fig. 7. Test case 1.1: time domain response of x_2 with co-simulation interface of sampling period 0.05 ms, and time delay of 0.5 ms

Therefore, for partitioning the circuit illustrated in Figure 4 decoupling point B should be used. In this case, due to the local fast mode that influences interface state, sampling period of the interface should be smaller than critical time constant $T_c = 0.72$ ms of the fast local mode (for delay of one time step). Figure 8 shows that for sampling period of 1.0 ms artificial dynamic is introduced in the response of x_2 (interface state dominantly influenced by fast mode). This dynamic leads to different shape of time response compared to monolithic one and therefore affects fidelity of the simulation.

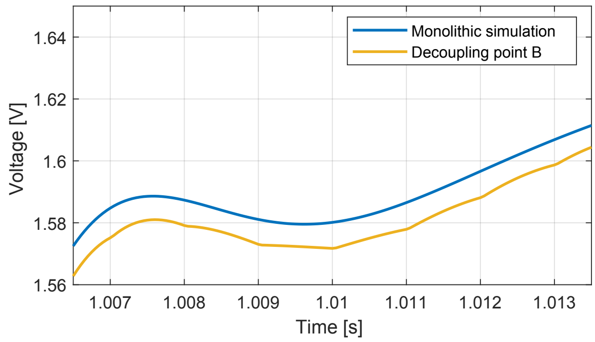

Fig. 8. Test case 1.1: time domain response of x_2 with co-simulation interface of sampling period 1.0 ms, and time delay of 0.05 ms

2) Test case 1.2: Parameters of the circuit for the test case 1.2 and resulting modes are given in the Table II. The selected parameters result in the system with one fast λ_1 mode and one slow $\lambda_{2,3}$ mode.

TABLE II
PARAMETERS FOR TEST CASE 1.2

Test case	Parameters	Modes		
		eigenvalues	T_c [ms]	type
1.2	$R_1 = 1.0\Omega$ $L_1 = 1.0mH$ $C = 10.0mF$	$\lambda_1 = -889.11$	0.56	fast
	$R_2 = 1.0\Omega$ $L_2 = 10.0mH$	$\lambda_{2,3} = -104.45 \pm i \cdot 106.66$	4.7	slow

Participation matrix for test case 1.2 is:

$$P = \begin{bmatrix} 1.1376 & -0.0688 + 0.0142 \cdot i & -0.0688 - 0.0142 \cdot i \\ -0.1399 & 0.5699 + 0.0196 \cdot i & 0.5699 - 0.0196 \cdot i \\ 0.0022 & 0.4989 - 0.0338 \cdot i & 0.4989 + 0.0338 \cdot i \end{bmatrix}$$

As indicated in the participation matrix, the state x_1 is mainly influenced by the fast λ_1 mode. The slow $\lambda_{2,3}$ mode is associated with the states x_2 and x_3 while its influence on the state x_1 can be neglected. The described participation of modes in the states for test case 1.2 is illustrated in the Figure 9.

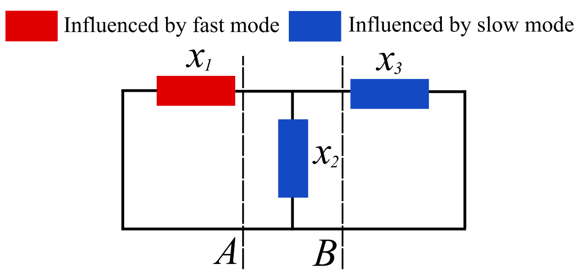

Fig. 9. Test case 1.2 - Participation of modes in states

As a result of analysis based on dynamics of modes and participation matrix, characteristics of decoupling point A are given below:

- Partitioned system is loosely coupled
- There is a local fast mode with critical time constant of $T_c = 0.56$ ms that influences interface state (interface quantity)
- Recommended limit for maximum sampling period to avoid artificial dynamic in system response is $T_c = 0.56$ ms (for one time step delay)

Characteristics of decoupling point B:

- Partitioned system is not loosely coupled
- There is one coupling mode that is slow

In case of decoupling points with characteristics as given above, additional analysis is required to select the point for system partitioning that provides higher degree of fidelity. Although decoupling point A results in loosely coupled system, system partitioned with decoupling point B has only a slow coupling mode. Figure 10 confirms that there is no clear difference in fidelity of the two decoupling points.

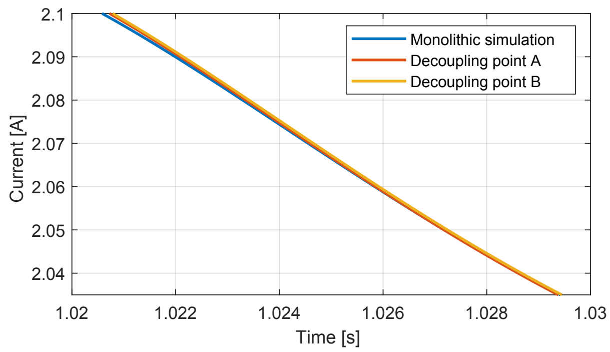

Fig. 10. Test case 1.2: time domain response of x_1 with co-simulation interface of sampling period 0.05 ms, and time delay of 0.05 ms

Furthermore, empirical analysis indicates that decoupling point B characterized with a slow coupling mode can be a better choice compared to a decoupling point A that results in loosely coupled system, especially if time delay is increased. Similarly, increasing sampling period deteriorates fidelity for decoupling point A with respect to decoupling point B. Note that the above conclusions cannot be derived only based on analysis of participation matrix, since in that case a loosely coupled point would be selected. Therefore, eigenvalues analysis is needed in addition to participation matrix assessment to select a suitable decoupling point.

B. Test case 2

The simple electrical circuit utilized for test case 1 is extended with an RC branch and used as test case 2 to further demonstrate the method described in Section III. Figure 11 illustrates the circuit selected for test case 2. Simulation of the circuit was running 0.5 s, and transient process was initiated with changing the input V_1 from 5 to 4 V at 0.2 s.

Parameters of the circuit for the test case 2 and resulting modes are given in the Table III. The system has one fast $\lambda_{1,2}$ mode and one slow $\lambda_{3,4}$ mode.

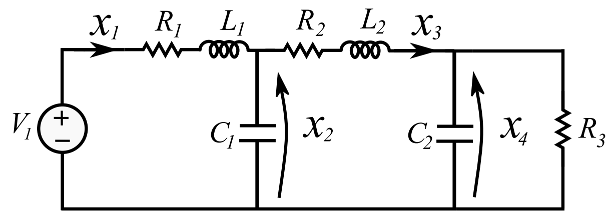

Fig. 11. Test Case 2 - simple electrical circuit

TABLE III
PARAMETERS FOR TEST CASE 2

Test case	Parameters	Modes: eigenvalues	Modes: T_c [ms]	Modes: type
2	$R_1 = 1.0\Omega$, $L_1 = 1.0mH$, $C_1 = 1.0mF$, $R_2 = 1.0\Omega$	$\lambda_{1,2} = -451.6 \pm i \cdot 904.79$	0.69	fast
	$L_2 = 10.0mH$, $C_2 = 10.0mF$, $R_3 = 1.0\Omega$	$\lambda_{3,4} = -148.4 \pm i \cdot 85.53$	3.35	slow

Participation matrix for test case 2 is:

$$P = \begin{bmatrix} 0.4323 + 0.3231 \cdot i & 0.4323 - 0.3231 \cdot i & 0.0677 + 0.0277 \cdot i & 0.0677 - 0.0277 \cdot i \\ 0.5446 - 0.2617 \cdot i & 0.5446 + 0.2617 \cdot i & -0.0446 - 0.0297 \cdot i & -0.0446 + 0.0297 \cdot i \\ 0.0234 - 0.0608 \cdot i & 0.0234 + 0.0608 \cdot i & 0.4766 + 0.2773 \cdot i & 0.4766 - 0.2773 \cdot i \\ -0.0003 - 0.0006 \cdot i & -0.0003 + 0.0006 \cdot i & 0.5003 - 0.2752 \cdot i & 0.5003 + 0.2752 \cdot i \end{bmatrix} \tag{9}$$

Participation matrix indicates that fast mode $\lambda_{1,2}$ is mainly associated with states x_1 and x_2, while its relation with the states x_3 and x_4 can be neglected. Similarly, slow mode $\lambda_{3,4}$ has negligible influence on states x_1 and x_2 and from participation matrix it can be concluded that it mainly influences states x_3 and x_4. Figure 3 illustrates the described participation of modes in states. As illustrated in the Figure 3, there are three possible decoupling points A, B and C.

Following the methodology described in the section III, the main characteristics of decoupling point A are listed below:

- Partitioned system is not loosely coupled
- There is a fast coupling mode with critical time constant of $T_c = 0.69$ ms
- Recommended limit for maximum time delay to ensure stability is $T_c = 0.69$ ms (if sampling period is equal to one time step — 0.05 ms)
- Recommended limit for maximum sampling period to ensure stability is $1.1 \cdot T_c = 0.759$ ms (if time delay is equal to one time step — 0.05 ms)

The main characteristics of decoupling point B are given below:

- Partitioned system is loosely coupled
- There is a local fast mode with critical time constant of $0.69ms$ that influences interface state (interface quantity)

- Recommended limit for maximum sampling period to avoid artificial dynamic in system response is $T_c = 0.69$ ms (if time delay is equal to one time step — 0.05 ms)

Characteristics of decoupling point C:

- Partitioned system is not loosely coupled
- There is one coupling mode that is slow

Decoupling point A is definitely not a suitable candidate since the partitioned system is characterized with fast coupling mode with critical time constant of $T_c = 0.69$ ms. Results indicate that for decoupling point A, with sampling period of one time step, system is stable for 0.5 ms delay and unstable for delay of 0.75 ms. The decoupling point A results in significantly lower degree of simulation fidelity in comparison to decoupling points B and C, which can be clearly concluded based on response of state x_2 illustrated in the Figure 12. If the time delay is equal to one time step (0.05 ms) and sampling period is increased, empirical results show that the stability margin is slightly increased from $T_c = 0.69$ ms to $1.1 \cdot T_c = 0.759$ ms. The results confirm that the system is stable for 0.75 ms, but unstable for 1 ms.

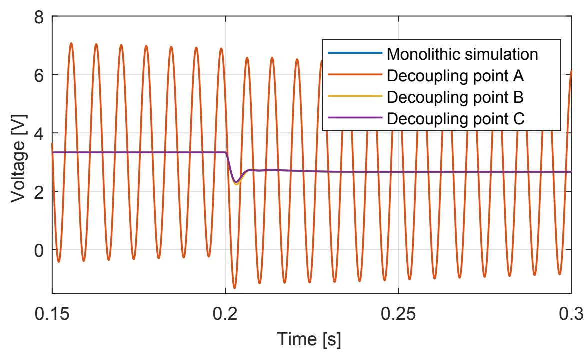

Fig. 12. Test case 2: time domain response of x_2 with co-simulation interface of sampling periods of 0.05 ms and time delay of 0.5 ms

In the Figure 13 the time response of x_2 following the disturbance can be observed for interface time delay of 0.05 ms and different sampling periods. For sampling period of 0.75 ms artificial dynamic is significantly visible.

As it is explained in the section III additional analysis is required to determine the decoupling point that provides higher degree of fidelity between B and C. Although decoupling point B results in a loosely coupled system, system partitioned with decoupling point C has only a slow coupling mode. Empirical analysis implies that decoupling point C may result in a higher degree of simulation fidelity compared to the decoupling point B, in particular for larger values of sampling period and time delay.

Similarly as for test case 1.2, note that the above conclusions could not be provided only based on observing participation matrix. Namely, participation matrix suggests that decoupling point B is suitable as it results in a loosely coupled system. However, additional analysis of eigenvalues suggests that decoupling point C should be considered as well as it has only

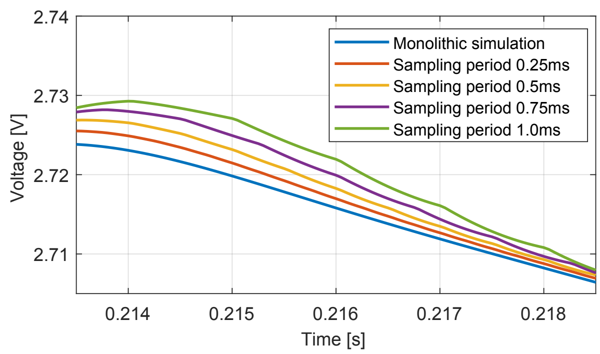

Fig. 13. Test case 2: time domain response of x_2 with co-simulation interface of time delay of 0.05 ms and sampling periods of 0.25 ms, 0.5 ms, 0.75 ms, and 1.0 ms for decoupling point B

one slow coupling mode and there are not local fast modes that directly influence interface quantity.

V. Conclusion

Our paper focuses on analyzing decoupling points for system partitioning regrading fidelity of the simulations taking into account time delay and sampling period between two decoupled subsystems. It is important to note that there may be another factors that influence decoupling point selection (system partitioning) such as determined point of coupling for device under test. The methodology proposed in this work suggests that in addition to analyzing participation matrix and exploiting points of loose coupling, dynamics of the interface quantities and coupling modes should be considered. Improved approach will be implemented on AC systems in order to investigate results of this analysis on the model of the real test benches.

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